Example 3: **Find the $\sqrt[6]{2}$ correct to eight decimal places by using Newton's method.**

Solution:

Let $x = \sqrt[6]{2}$

So, $x^6 - 2 = 0$

Given function is

$f(x) = x^6 - 2 = 0$

So, $f'(x) = 6x^5$

By Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^6 - 2}{6x_n^5}$$

Since, we know $1 < \sqrt[6]{2} < 2$.

So, choose the initial approximation, $x_1 = 1$ then the other approximations are

$$x_2 = 1 - \frac{(1-2)}{6} = 1.16666667$$

$$x_3 = 1.6666667 - \frac{(1.6666667)^6 - 2}{6(1.666667)^5} = 1.2644368 \quad \text{(correct to 0 decimal place)}$$

$$x_4 = 1.2644368 - \frac{(1.2644368)^6 - 2}{6(1.26444368)^5} = 1.2249707 \quad \text{(correct to 1 decimal place)}$$

$$x_5 = 1.2249707 - \frac{(1.2249707)^6 - 2}{6(1.2249707)^5} = 1.12246205 \quad \text{(correct to 0 decimal place)}$$

$$x_6 = 1.2246205 - \frac{(1.2249707)^6 - 2}{6(1.2249070)^5} = 1.12246205 \quad \text{(correct to 8 decimal places)}$$

Thus, the sixth approximation shows the root of f(x) correct to 8 decimal places. And, the root of f(x) is given by 6^{th} approximation is 1.12246205.